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On Transceiver Beamformer Design for Multi-Source Multi-Destination Wireless Networks

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Abstract— **In this paper, we consider the problem of designing transceiver beamforming vectors for multi-source multi-destination (MSMD) wireless networks such that the transmission power of each source is minimized while the signal-to-interference plus noise ratio (SINR) requirements of all source-destination pairs are satisfied. We propose an efficient iterative algorithm to design the transceiver beamforming vectors and address the convergence of the algorithm. We determine a necessary condition as well as a sufficient condition for the algorithm to converge to a generalized Nash equilibrium solution. Especially, if each destination has only one antenna, we obtain a necessary and sufficient condition for the algorithm to converge to a unique generalized Nash equilibrium solution. Simulation results show that the proposed iterative algorithm has higher probability of convergence compared to an iterative waterfilling (IWF) approach. For example, for a system with three source-destination pairs and SINR requirement of 2dB, the probability of convergence is 84% with the proposed algorithm while it is only 66% with the IWF algorithm.**

Index Terms:[1] **Multi-source multi-destination (MSMD) wireless network, MIMO interference channel, transceiver beamforming, game theory, generalized Nash equilibrium.**

I. Introduction

Cognitive radio network is a promising technology in aim to improve spectrum utilization in wireless networks where secondary users may share primary users' spectrum resource without causing significant interference [1], [2]. Cognitive radio channel model can be characterized as interference channels with multi-sources and multi-destinations (MSMD). To improve performance of interference channels, the use of multiple-input multiple-output (MIMO) technology turns out to be an effective approach, in which each source and destination is equipped with multiple antennas. In recent years, the design of optimal transceiver beamformer for MIMO interference channel has attracted great interest [3]-[5], where beamforming vectors were designed to either minimize the total power consumption of the system or maximize Quality of Service (QoS) of the system. Various QoS criteria have been considered in the literature (see for example [4], [5], and the references therein), including the minimization of the weighted sum of mean-square error (MSE), the maximization of the weighted sum of user data rates, and the minimization of the total transmission power of all users.

Game theory approach [6] has been utilized to design transceiver beamformer for MSMD networks where the QoS of each source-destination pair is maximized subject to power constraint of each source [7]-[9]. In [7], an iterative algorithm was proposed to design beamforming vectors and it was shown that the algorithm converges to a *Nash Equilibrium (NE)* point [6] if the channel matrices are of full column rank. In [8], a beamformer design algorithm was proposed for nonsingular channel matrices and the impact of multiuser interference on the convergence of the algorithm was discussed. Furthermore, in [9], the authors extended the algorithm to accommodate arbitrary channel matrices and provided a unified view by proposing an iterative waterfilling (IWF) algorithm.

In this paper, we consider the problem of designing transceiver beamforming vectors for MSMD MIMO systems such that the transmission power of each source is minimized while the signal-to-interference plus noise ratio (SINR) requirement of each source-destination pair is satisfied. We know that for a single-source single-destination MIMO system, the minimization of transmission power with a QoS constraint is equivalent to the optimization of QoS performance with a power constraint due to the duality of the two optimization problems [10]. However, for a MSMD MIMO system, the minimization of each source-destination pair's power with QoS constraints is no longer equivalent to the optimization of QoS performance with power constraints. In this work, we propose an iterative algorithm to design the transceiver beamforming vectors which has a higher probability of convergence than an IWF design approach. We also determine a necessary condition as well as a sufficient condition for the algorithm to converge to a generalized Nash Equilibrium solution. Especially, if each destination has only one antenna, a necessary and sufficient condition is determined for the algorithm to converge to the unique generalized Nash Equilibrium solution. Numerical and simulation results are provided to illustrate the proposed algorithm and the theoretical development.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^\sharp$ represent the transpose, the Hermitian and the Moore-Penrose pseudoinverse of a matrix, respectively. $\rho(\cdot)$ is defined as the spectral radius of a matrix [13]. A matrix $\mathbf{B}$ or a vector $\mathbf{b}$ is called *non-negative (positive)* if each component of the matrix or vector is non-negative (positive), denoted as $\mathbf{B} \geq 0 (> 0)$ or $\mathbf{b} \geq 0 (> 0)$, respectively.

II. System Model and Problem Formulation

We consider a MSMD wireless network which consists of K source-destination pairs, where each source $S_k, k = 1, 2, \cdots, K$, transmits information to its intended destination $D_k, k = 1, 2, \cdots, K$, respectively. We assume that each

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source has M transmit antennas and each destination has N receive antennas. Let b_k be the transmitted information symbols at source S_k. Then the signal sent by the source S_k can be expressed as

$$\mathbf{s}_k = \mathbf{w}_k b_k,$$

where $\mathbf{w}_k \in \mathbb{C}^{M\times 1}$ is a beamforming vector at source S_k, and $E\{|b_k|^2\} = 1$. So, the average transmission power at source S_k is

$$E\{||\mathbf{w}_k b_k||^2\} = \mathbf{w}_k^H \mathbf{w}_k. \tag{1}$$

The received signal at destination D_k can be modeled as

$$\mathbf{y}_k \;=\; \mathbf{H}_{kk}\mathbf{w}_k b_k + \sum_{l=1,l\neq k}^{K} \mathbf{H}_{lk}\mathbf{w}_l b_l + \mathbf{n}_k, \tag{2}$$

where $\mathbf{H}_{kk} \in \mathbb{C}^{N\times M}$ is the channel matrix between source S_k and destination D_k, $\mathbf{H}_{lk}, (l \neq k)$, is the channel matrix between source S_l and destination D_k. The noise vector $\mathbf{n}_k$ at destination D_k is assumed to be an N-dimensional white Gaussian noise vector with mean zero and variance $\sigma_k^2\mathbf{I}_N$. Without loss of generality, we assume $\sigma_k^2 = 1$, $1 \leq k \leq K$.

At each destination D_k, a receiver beamforming vector $\mathbf{u}_k \in \mathbb{C}^{N\times 1}$ is applied to the received signal $\mathbf{y}_k$, then the resulting SINR at destination D_k can be represented by

$$\textbf{SINR}_k = \frac{\mathbf{w}_k^H\mathbf{H}_{kk}^H\mathbf{u}_k\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{w}_k}{\mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K}\mathbf{w}_l^H\mathbf{H}_{lk}^H\mathbf{u}_k\mathbf{u}_k^H\mathbf{H}_{lk}\mathbf{w}_l}, \tag{3}$$

in which the interference from other sources is treated as additive color noise.

For any given SINR requirement (e.g. QoS constraint) of each source-destination pair, we try to minimize the transmission power for each source. The optimization problem is formulated as follows:

$$\begin{cases} \texttt{minimize} & \mathbf{w}_k^H\mathbf{w}_k \\ \texttt{subject to} & \textbf{SINR}_k \geq \gamma_k \end{cases} \quad k = 1, 2, \cdots, K, \tag{4}$$

where γ_k is the SINR requirement of the source-destination pair (S_k, D_k). Without loss of generality, we assume that $\gamma_k = 1$ for all k, otherwise we can always obtain an equivalent problem by replacing $\mathbf{H}_{kk}$ with $\mathbf{H}_{kk}/\sqrt{\gamma_k}$. According to (3), the optimization problem can be written as

$$\begin{cases} \texttt{minimize} \;\; \mathbf{w}_k^H\mathbf{w}_k \qquad k = 1, 2, \cdots, K \\ \texttt{subject to} \\ \qquad \mathbf{w}_k^H\mathbf{G}_{kk}\mathbf{w}_k \geq \mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K}\mathbf{w}_l^H\mathbf{G}_{lk}\mathbf{w}_l, \end{cases} \tag{5}$$

where $\mathbf{G}_{lk} \triangleq \mathbf{H}_{lk}^H\mathbf{u}_k\mathbf{u}_k^H\mathbf{H}_{lk}$ for $1 \leq l, k \leq K$.

III. Beamforming Design and Convergence of the Algorithm

A. Transceiver Beamformer Design

For simplicity of notation, let us denote the transmitter beamformer set as $\mathbf{W} = (\mathbf{w}_1, ..., \mathbf{w}_K)$ and the receiver beamformer set as $\mathbf{U} = (\mathbf{u}_1, ..., \mathbf{u}_K)$. First, we consider receiver beamforming design at each destination for any fixed transmitter beamformer set $\mathbf{W}$. Note that, at each destination D_k, the covariance matrix of the noise plus interference is given by

$$\mathbf{R}_k = \mathbf{I}_N + \sum_{l=1,l\neq k}^{K}\mathbf{H}_{lk}\mathbf{w}_l\mathbf{w}_l^H\mathbf{H}_{lk}^H, \tag{6}$$

which can be estimated at destination D_k based on the received signals. The receiver beamforming vector $\mathbf{u}_k$ that maximizes SINR_k at destination D_k is given by [4]

$$\mathbf{u}_k = \mathbf{R}_k^{-1}\mathbf{H}_{kk}\mathbf{w}_k. \tag{7}$$

The resulting maximal SINR_k is

$$\textbf{SINR}_k = \mathbf{w}_k^H\mathbf{H}_{kk}^H\mathbf{R}_k^{-1}\mathbf{H}_{kk}\mathbf{w}_k. \tag{8}$$

Now let us consider beamforming design at the transmitter side. At each source S_k, with given receiver beamforming vector $\mathbf{u}_k$ and fixed other users' transmit beamformer vectors, to maximize $\mathbf{w}_k^H\mathbf{G}_{kk}\mathbf{w}_k$ in (5), the optimal transmit beamformer $\mathbf{w}_k$ at source S_k should be parallel to the vector $\mathbf{H}_{kk}^H\mathbf{u}_k$. Let us denote the normalized vector of $\mathbf{H}_{kk}^H\mathbf{u}_k$ as $\mathbf{v}_k$ and $\lambda_k \triangleq \mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k$, then we have

$$\mathbf{v}_k = \frac{1}{\sqrt{\lambda_k}}\mathbf{H}_{kk}^H\mathbf{u}_k. \tag{9}$$

To minimize the transmission power $\mathbf{w}_k^H\mathbf{w}_k$ in (5), by Lagrangian method, the optimal transmit beamformer $\mathbf{w}_k$ should follow with the equality of the constraint, i.e., $\mathbf{w}_k^H\mathbf{G}_{kk}\mathbf{w}_k = \mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K}\mathbf{w}_l^H\mathbf{G}_{lk}\mathbf{w}_l$, and with the notation in (9), the corresponding transmission power $\mathbf{w}_k^H\mathbf{w}_k$ is given by

$$||\mathbf{w}_k||^2 = \frac{1}{\lambda_k}(\mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K}\mathbf{w}_l^H\mathbf{G}_{lk}\mathbf{w}_l). \tag{10}$$

Therefore, for any source S_k, $1 \leq k \leq K$, the optimal transmit beamformer $\mathbf{w}_k$ that minimizes its transmission power subject to the QoS constraints should satisfy the following equation:

$$\mathbf{w}_k = \left(\frac{1}{\lambda_k}(\mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K}\mathbf{w}_l^H\mathbf{G}_{lk}\mathbf{w}_l)\right)^{1/2}\mathbf{v}_k,\;\; 1 \leq k \leq K. \tag{11}$$

According to (6), and (9), the optimal transmit beamformer $\mathbf{w}_k$ at each source S_k can be represented as

$$\mathbf{w}_k = \frac{\sqrt{\mathbf{u}_k^H\mathbf{R}_k\mathbf{u}_k}}{\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k}\mathbf{H}_{kk}^H\mathbf{u}_k, \quad 1 \leq k \leq K, \tag{12}$$

in which the covariance matrix $\mathbf{R}_k$ is defined in (6) that can be estimated at destination D_k.

From (10) and (11), we can see that the power minimization problem can be formulated as a fixed-point problem as follows. Denote $\mathbf{x} \triangleq (x_1, x_2, \cdots, x_K)^T$, where $x_k = ||\mathbf{w}_k||^2$ for any $k = 1, 2, \cdots, K$, then the equation (10) can be written as $x_k = \frac{1}{\lambda_k}(\mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K}\mathbf{v}_l^H\mathbf{G}_{lk}\mathbf{v}_l x_l)$ for any $1 \leq k \leq$

K. Let us define a mapping $\mathbf{F} : R_+^K \rightarrow R_+^K$ as, $\mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_K(\mathbf{x}))^T$, where

$$f_k(\mathbf{x}) = \frac{1}{\lambda_k}(\mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K} \mathbf{v}_l^H\mathbf{G}_{lk}\mathbf{v}_l x_l), \ 1 \leq k \leq K. \quad (13)$$

Then, to find the optimal transmit beamformer vectors $(\mathbf{w}_1, \cdots, \mathbf{w}_K)$ that satisfy the K equations in (10) is equivalent to solve the following fixed-point problem:

$$\mathbf{x} = \mathbf{F}(\mathbf{x}). \quad (14)$$

Furthermore, let us denote vector $\mathbf{a} \triangleq (a_1, a_2, ..., a_K)^T$ where $a_k = \frac{\mathbf{u}_k^H\mathbf{u}_k}{\lambda_k}$, and matrix $\mathbf{B}$ with components

$$\mathbf{B}_{kl} \triangleq \begin{cases} \frac{\mathbf{v}_l^H\mathbf{G}_{lk}\mathbf{v}_l}{\lambda_k}, & 1 \leq k \neq l \leq K; \\ 0, & 1 \leq k = l \leq K, \end{cases} \quad (15)$$

then, the fixed-point problem in (14) can be written as

$$\mathbf{x} = \mathbf{F}(\mathbf{x}) = \mathbf{a} + \mathbf{B}\mathbf{x}. \quad (16)$$

Note that, according to (9), the components of the vector $\mathbf{a}$ can be specified as

$$a_k = \frac{\mathbf{u}_k^H\mathbf{u}_k}{\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k}, \quad 1 \leq k \leq K, \quad (17)$$

and the components of the matrix $\mathbf{B}$ can be specified as

$$\mathbf{B}_{kl} = \frac{\mathbf{u}_l^H\mathbf{H}_{ll}\mathbf{H}_{lk}^H\mathbf{u}_k\mathbf{u}_k^H\mathbf{H}_{lk}\mathbf{H}_{ll}^H\mathbf{u}_l}{\mathbf{u}_l^H\mathbf{H}_{ll}\mathbf{H}_{ll}^H\mathbf{u}_l\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k}, \quad 1 \leq k \neq l \leq K. \quad (18)$$

We observe that if $\mathbf{I}-\mathbf{B}$ is invertible, the fixed-point problem in (16) has a unique solution which is given by[2]

$$\mathbf{x} = (\mathbf{I} - \mathbf{B})^{-1}\mathbf{a}. \quad (19)$$

If the solution $\mathbf{x}$ is positive, then the optimal transmitter beamforming vectors $(\mathbf{w}_1, \cdots, \mathbf{w}_K)$ are determined as

$$\mathbf{w}_k = \sqrt{x_k}\mathbf{v}_k, \quad 1 \leq k \leq K, \quad (20)$$

where $\mathbf{v}_k$ is specified in (9), and the corresponding optimal receiver beamforming vectors $(\mathbf{u}_1, \cdots, \mathbf{u}_K)$ can be determined based on (7) accordingly.

We may view the MSMD beamformer design problem as a noncooperative Nash equilibrium game [6], in which each source-destination pair is an active player and they compete each other to minimize their own transmission power. The payoff of each player is its own transmission power and the strategy of each player is to act aggressively and selfishly to adjust its transceiver beamformer vectors $\mathbf{u}_k$ and $\mathbf{w}_k$ according to (7) and (12), respectively. We can see that the strategy of each player depends on others', so it is a *generalized Nash equilibrium* (GNE) game [6] and the corresponding equilibrium is called a GNE solution. A transceiver beamformer set $(\mathbf{W}, \mathbf{U})$ is a GNE solution if for each source-destination pair (S_k, D_k), given the other pairs' transceiver beamformers, the transmission power of source S_k cannot be further reduced by optimizing the transceiver beamformer design of this pair. The resulting beamformer set $(\mathbf{W}, \mathbf{U})$ satisfy (7) and (12) for each source-destination pair, i.e.,

$$\begin{aligned} \mathbf{u}_k &= \mathbf{R}_k^{-1}\mathbf{H}_{kk}\mathbf{w}_k, \quad 1 \leq k \leq K, & (21)\\ \mathbf{w}_k &= \frac{\sqrt{\mathbf{u}_k^H\mathbf{R}_k\mathbf{u}_k}}{\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k}\mathbf{H}_{kk}^H\mathbf{u}_k, \quad 1 \leq k \leq K. & (22) \end{aligned}$$

Note that, from (21) and (22), we have

$$\begin{aligned} \mathbf{u}_k^H\mathbf{R}_k\mathbf{u}_k &= \mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{w}_k \\ &= \mathbf{u}_k^H\mathbf{H}_{kk}\frac{\sqrt{\mathbf{u}_k^H\mathbf{R}_k\mathbf{u}_k}}{\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k}\mathbf{H}_{kk}^H\mathbf{u}_k \\ &= \sqrt{\mathbf{u}_k^H\mathbf{R}_k\mathbf{u}_k}, \end{aligned}$$

which implies that $\mathbf{u}_k^H\mathbf{R}_k\mathbf{u}_k = 1$. So, the iterative process in (22) can be replaced by

$$\mathbf{w}_k = \frac{\mathbf{H}_{kk}^H\mathbf{u}_k}{\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k}, \quad 1 \leq k \leq K. \quad (23)$$

In the following, we propose an iterative algorithm to design transceiver beamformer set $(\mathbf{W}, \mathbf{U})$ that minimizes each source's transmission power while satisfying the SINR requirements of all source-destination pairs.

Algorithm:

- Step 1: Initialize the transceiver beamformer set $(\mathbf{W}, \mathbf{U})$ with random vectors;
- Step 2: Determine matrix $\mathbf{B}$ according to (18), and set $\mathbf{x} = (\mathbf{I} - \mathbf{B})^\sharp\mathbf{a}$, where $\mathbf{a}$ is given by (17);
- Step 3: `if` $\mathbf{x} \geq 0$, `then`
  update $\mathbf{w}_k^{(n)}$ using (20) for all $1 \leq k \leq K$;
  `else`
  update $\mathbf{w}_k^{(n)}$ using (12) for k with $x_k < 0$;
  `end;`
- Step 4: `if` $||\mathbf{w}_k^{(n+1)} - \mathbf{w}_k^{(n)}||_2^2 \leq \epsilon$ for all $1 \leq k \leq K$, `then`
  output the transceiver beamformer vectors $\mathbf{w}_k^{(n)}$ and $\mathbf{u}_k^{(n)}$ for all $1 \leq k \leq K$.
  `else`
  update $\mathbf{u}_k^{(n)}$ by (7), and goto Step 2;
  `end;`
- Step 5: `if` the maximal number of iterations reaches, `then`
  output failure message and stop;
  `else` goto Step 1.

The convergence of the above iterative algorithm is addressed in the next subsection. We note that an alternative iterative waterfilling (IWF) approach [9] may be utilized to design the transmit beamformer set $\mathbf{W}$. In this case, the transmit beamformer for each source $\mathbf{w}_k$ is chosen as the eigenvector of $\mathbf{H}_{kk}^H\mathbf{R}_k^{-1}\mathbf{H}_{kk}$ corresponding to the maximal eigenvalue λ_{max}, which results in $\mathbf{w}_k^H\mathbf{w}_k = 1/\lambda_{max}$. As we will see in the simulation results, our proposed algorithm has a higher probability of converging to a GNE solution than the IWF approach.

[2]Note that if $\mathbf{I} - \mathbf{B}$ is not invertible, we may use the Moore-Penrose pseudoinverse $(\mathbf{I} - \mathbf{B})^\sharp$ to represent the solution $\mathbf{x}$ in (19).

B. Condition to the Convergence of the Algorithm

In this subsection, we determine a necessary condition as well as a sufficient condition for the convergence of the proposed iterative algorithm. We first develop two lemmas which are critical to obtain the main result.

Lemma 1: For a matrix $\mathbf{B} \geq 0$ and a vector $\mathbf{a} > 0$, the necessary and sufficient condition for equation $(\mathbf{I}-\mathbf{B})\mathbf{x} = \mathbf{a}$ to have a unique and non-negative solution is that the spectral radius $\rho(\mathbf{B})$ is less than 1, where $\rho(\mathbf{B}) = max\{|\lambda_i|\}$ and λ_i's are eigenvalues of $\mathbf{B}$.

Proof: If $\rho(\mathbf{B}) < 1$, we know that both $\mathbf{B}$ and $\mathbf{I}-\mathbf{B}$ are invertible [13]. Moreover, since $(\mathbf{I}-\mathbf{B})^{-1} = \sum_{n=0}^{\infty} \mathbf{B}^n$, which is non-negative with $\mathbf{B} \geq 0$, thus with positive $\mathbf{a}$, the equation $(\mathbf{I}-\mathbf{B})\mathbf{x} = \mathbf{a}$ has a solution $\mathbf{x} = (\mathbf{I}-\mathbf{B})^{-1}\mathbf{a}$ which is unique and non-negative.

On the other hand, for any non-negative matrix $\mathbf{B}$, according to the Perron-Frobenis theorem [13], there exists a non-negative vector $\mathbf{d} \neq \mathbf{0}$ such that $\mathbf{B}^T\mathbf{d} = \rho(\mathbf{B})\mathbf{d}$. So, if the equation $(\mathbf{I}-\mathbf{B})\mathbf{x} = \mathbf{a}$ has a non-negative solution, we have

$$(1-\rho(\mathbf{B}))\mathbf{d}^T\mathbf{x} = \mathbf{d}^T(\mathbf{I}-\mathbf{B})\mathbf{x} = \mathbf{d}^T\mathbf{a} > 0.$$

Since $\mathbf{d}^T\mathbf{x} \geq 0$, so $1-\rho(\mathbf{B})$ must be positive. Thus, we have the necessary condition $\rho(\mathbf{B}) < 1$. ∎

Lemma 2: If a matrix $\mathbf{B} \in \mathbb{R}^{K\times K}$ has the following form:

$$\mathbf{B} = \begin{cases} b_{kl} \geq 0, & 1 \leq k \neq l \leq K; \\ b_{kk} = 0, & 1 \leq k \leq K, \end{cases}$$

then the spectral radius of $\mathbf{B}$ is less than 1, i.e. $\rho(\mathbf{B}) < 1$, if and only if there exist K positive values $x_1, x_2, \cdots, x_K$ such that

$$\frac{1}{x_k}\sum_{l=1,l\neq k}^{K} \mathbf{B}_{kl}x_l < 1, \quad 1 \leq k \leq K. \tag{24}$$

Proof: If $\rho(\mathbf{B}) < 1$, then by Lemma 1, for any vector $\mathbf{a} > 0$, there exists a unique and non-negative vector $\mathbf{x} = (x_1, x_2, \cdots, x_K)$ that satisfies the equation $(\mathbf{I}-\mathbf{B})\mathbf{x} = \mathbf{a}$. In this case, we have $(\mathbf{I}-\mathbf{B})\mathbf{x} > 0$, which leads to the condition in (24). Conversely, if the condition in (24) holds, which is equivalent to $(\mathbf{I}-\mathbf{B})\mathbf{x} > 0$, then there exists a positive vector $\mathbf{a}$ such that $(\mathbf{I}-\mathbf{B})\mathbf{x} = \mathbf{a}$, which means that the equation has a non-negative solution $\mathbf{x}$. According to Lemma 1 again, we have $\rho(\mathbf{B}) < 1$, which completes the proof. ∎

Based on Lemmas 1 and 2, we are able to determine in the following a necessary condition for the proposed algorithm to converge to a GNE solution.

Theorem 1: A necessary condition for the proposed algorithm to converge to a GNE solution is that there exists a vector set $\{\tilde{\mathbf{u}}_k \in \mathbb{C}^{N\times 1} : ||\tilde{\mathbf{u}}_k||^2 = 1, 1 \leq k \leq K\}$ such that

$$\rho(\mathbf{CD}^2) < 1, \tag{25}$$

where the matrices $\mathbf{C}$ and $\mathbf{D}$ are given by

$$\mathbf{C}_{kl} \triangleq \begin{cases} ||\tilde{\mathbf{u}}_k^H \mathbf{H}_{lk}\mathbf{H}_{ll}^H\tilde{\mathbf{u}}_l||^2, & 1 \leq k \neq l \leq K; \\ 0, & 1 \leq k = l \leq K, \end{cases} \tag{26}$$

$$\mathbf{D}_{kl} \triangleq \begin{cases} \frac{1}{\tilde{\mathbf{u}}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\tilde{\mathbf{u}}_k}, & 1 \leq k = l \leq K; \\ 0, & 1 \leq k \neq l \leq K. \end{cases} \tag{27}$$

Proof: If a GNE solution exists, it means that there exists a transceiver beamforming vector set $(\mathbf{W}, \mathbf{U})$ satisfying the two recursive equations (21) and (22). Furthermore, it implies that the fixed-point problem in (16) has a solution of $x_k = \mathbf{w}_k^H\mathbf{w}_k$, $1 \leq k \leq K$, where $\mathbf{w}_k$ is given in (22) or equivalently in (23). Following the expression in (23), the solution x_k can be specified as

$$x_k = \frac{1}{\mathbf{u}_k^H\mathbf{H}_{kk}\mathbf{H}_{kk}^H\mathbf{u}_k}, \quad 1 \leq k \leq K. \tag{28}$$

By substituting the above solution into the equation in (16), we have

$$\mathbf{u}_k^H\mathbf{u}_k + \sum_{l=1,l\neq k}^{K} \frac{\mathbf{u}_l^H\mathbf{H}_{ll}\mathbf{H}_{lk}^H\mathbf{u}_k\mathbf{u}_k^H\mathbf{H}_{lk}\mathbf{H}_{ll}^H\mathbf{u}_l}{(\mathbf{u}_l^H\mathbf{H}_{ll}\mathbf{H}_{ll}^H\mathbf{u}_l)^2} = 1,\ 1 \leq k \leq K. \tag{29}$$

Denote $\mathbf{q} \triangleq (q_1, ..., q_K)^T$, where $q_k = \frac{1}{\mathbf{u}_k^H\mathbf{u}_k}$, and let $\tilde{\mathbf{u}}_k = \sqrt{q_k}\mathbf{u}_k$ for $1 \leq k \leq K$, then $||\tilde{\mathbf{u}}_k||^2 = 1$, and with the matrices defined in (26) and (27), the equation in (29) can be written as

$$\frac{1}{q_k} + \sum_{l=1,l\neq k}^{K} \frac{\mathbf{C}_{kl}\mathbf{D}_{ll}^2 q_l}{q_k} = 1, \quad 1 \leq k \leq K, \tag{30}$$

which leads to a matrix representation as follows:

$$(\mathbf{I}-\mathbf{CD}^2)\mathbf{q} = \mathbf{1}. \tag{31}$$

According to Lemma 1, the existence of the above equation implies that the spectrum radius of the matrix $\mathbf{CD}^2$ must be less than 1, i.e. $\rho(\mathbf{CD}^2) < 1$. Thus, we prove the theorem completely. ∎

For some channel conditions, if the necessary condition in Theorem 1 is not satisfied, the proposed iterative algorithm will diverge. For example, let us consider a system with two source-destination pairs ($K = 2$) with $M = 2$ and $N = 1$. If the channels are given by

$$\begin{aligned} \mathbf{H}_{11} &= (0.1078 + 0.7184i, \quad 0.1189 + 0.7391i), \\ \mathbf{H}_{21} &= (-3.1297 - 1.6945i, \quad -1.2949 + 3.0532i), \\ \mathbf{H}_{12} &= (-4.2857 + 1.8133i, \quad 4.0810 - 3.0028i), \\ \mathbf{H}_{22} &= (1.0186 + 1.0264i, \quad 0.2113 + 0.1098i), \end{aligned}$$

then for any unit vector $\tilde{\mathbf{u}}_k, k = 1, 2$, the matrix $\mathbf{CD}^2$ is

$$\mathbf{CD}^2 = \begin{pmatrix} 0 & 21.7475 \\ 22.0352 & 0 \end{pmatrix}. \tag{32}$$

The spectral radius of the matrix $\mathbf{CD}^2$ is $\rho(\mathbf{CD}^2) = 21.8909$ which is larger than 1. In this case, both the proposed algorithm and the IWF algorithm diverge as shown in Fig.1.

In the following, we obtain a sufficient condition for the proposed algorithm to converge to a GNE solution. Before we present the theorem, we introduce a lemma and its proof is omitted due to space limitation.

Lemma 3: For a $K \times K$ matrix $\mathbf{B} \geq 0$ and a K dimension vector $\mathbf{a} > 0$, if there is a non-negative vector $\mathbf{x} \triangleq (x_1, x_2, ..., x_K)^T$ such that $(\mathbf{I}-\mathbf{B})\mathbf{x} \geq \mathbf{a}$, then there exists a non-negative vector $\tilde{\mathbf{x}} \triangleq (\tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_K)^T$ such that $(\mathbf{I}-\mathbf{B})\tilde{\mathbf{x}} = \mathbf{a}$ and $\tilde{x}_k \leq x_k$ for all $1 \leq k \leq K$.

Theorem 2: The proposed algorithm converges to a GNE solution, if for any vector set $\{\tilde{\mathbf{u}}_k \in \mathbb{C}^{N\times 1} : ||\tilde{\mathbf{u}}_k||^2 = 1, 1 \leq k \leq K\}$, the following condition is satisfied

$$\rho(\mathbf{CD}^2) < 1, \tag{33}$$

where the matrices $\mathbf{C}$ and $\mathbf{D}$ are defined in (26) and (27), respectively.

Proof: For any vector set $\{\tilde{\mathbf{u}}_k \in \mathbb{C}^{N\times 1} : ||\tilde{\mathbf{u}}_k||^2 = 1, 1 \leq k \leq K\}$, we observe that the matrix $\mathbf{B}$ in (18) can be represented as $\mathbf{B} = \mathbf{DCD}$, where the matrices $\mathbf{C}$ and $\mathbf{D}$ are defined as in (26) and (27), respectively. So, we have

$$\rho(\mathbf{B}) = \rho(\mathbf{DCD}) = \rho(\mathbf{CD}^2). \tag{34}$$

If $\rho(\mathbf{CD}^2) < 1$, then $\rho(\mathbf{B}) < 1$. According to Lemma 1, the fixed-point problem in (16) always guarantees a non-negative solution for any receiver beamforming vector set $\mathbf{U}$. Let us denote $\mathbf{U}^{(n)} \triangleq (\mathbf{u}_1^{(n)}, \mathbf{u}_2^{(n)}, ..., \mathbf{u}_K^{(n)})$ and $\mathbf{W}^{(n)} \triangleq (\mathbf{w}_1^{(n)}, \mathbf{w}_2^{(n)}, ..., \mathbf{w}_K^{(n)})$, with $\mathbf{u}_k^{(n)}$ and $\mathbf{w}_k^{(n)}$ denoting the transceiver beamforming vectors of the k-th source-destination pair at n iteration of the proposed algorithm. At any iteration n, the transceiver beamforming vector set $(\mathbf{W}^{(n)}, \mathbf{U}^{(n)})$ is chosen to satisfy the equations in (16), or equivalently in (10). It implies, from (9) and (20), that the SINR of the k-th source-destination pair given by the transceiver beamforming vector set $(\mathbf{W}^{(n)}, \mathbf{U}^{(n)})$ is equal to its minimal requirement, i.e. $\text{SINR}_k(\mathbf{W}^{(n)}, \mathbf{U}^{(n)}) = 1$ for all $1 \leq k \leq K$. At $n+1$ iteration, receiver beamformer $\mathbf{U}^{(n+1)}$ is chosen to maximize SINR_k for given transmit beamformer $\mathbf{W}^{(n)}$, so $\text{SINR}_k(\mathbf{W}^{(n)}, \mathbf{U}^{(n+1)}) \geq 1$, which means there exists a transmit beamformer set $\tilde{\mathbf{W}} \triangleq (\tilde{\mathbf{w}}_1, \tilde{\mathbf{w}}_2, ..., \tilde{\mathbf{w}}_K)$ such that $\text{SINR}_k(\tilde{\mathbf{W}}, \mathbf{U}^{(n+1)}) = 1$ for all $1 \leq k \leq K$ and $||\tilde{\mathbf{w}}_k||^2 \leq ||\mathbf{w}_k^{(n)}||^2$, for all $1 \leq k \leq K$. For given receiver beamformer set $\mathbf{U}^{(n+1)}$, since $\mathbf{W}^{(n+1)}$ is the optimal transmit beamformer set which minimizes the transmission power $||\mathbf{w}_k||^2$ for all $1 \leq k \leq K$, so $||\mathbf{w}_k^{(n+1)}||^2 \leq ||\tilde{\mathbf{w}}_k||^2$ for all $1 \leq k \leq K$. Therefore we have $||\mathbf{w}_k^{(n+1)}||^2 \leq ||\mathbf{w}_k^{(n)}||^2$ for all $1 \leq k \leq K$. It means that the transmission power $||\mathbf{w}_k||^2$, which has lower bound 0, decreases at every iterative step for all $1 \leq k \leq K$, thus the proposed algorithm converges to a GNE solution. ■

Interestingly, based on Theorems 1 and 2, we are able to obtain a necessary and sufficient condition for the convergence of the iterative algorithm if each destination has only one antenna ($N = 1$). In this case, each receiver beamformer is a scalar, i.e. $\tilde{\mathbf{u}}_k = 1$ for $1 \leq k \leq K$, and the matrices in (26) and (27) are reduced to:

$$\mathbf{C}_{kl} = \begin{cases} ||\mathbf{H}_{lk}\mathbf{H}_{ll}^H||^2, & 1 \leq k \neq l \leq K; \\ 0, & 1 \leq k = l \leq K, \end{cases} \tag{35}$$

$$\mathbf{D}_{kl} = \begin{cases} \frac{1}{\mathbf{H}_{kk}\mathbf{H}_{kk}^H}, & 1 \leq k = l \leq K; \\ 0, & 1 \leq k \neq l \leq K, \end{cases} \tag{36}$$

which depend only on the channel conditions. Therefore, according to Theorems 1 and 2, the proposed iterative algorithm converges to a unique GNE solution if and only if $\rho(\mathbf{CD}^2) < 1$. We summarize the above discussion in the following result:

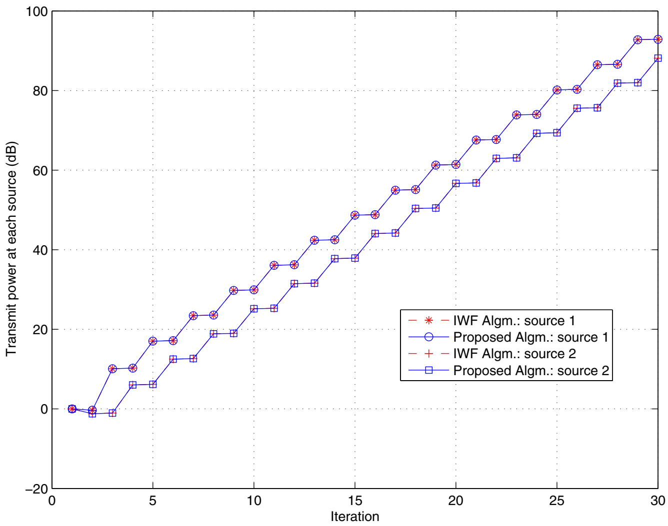

Fig. 1. Divergence case with $K = 2$ and $\rho(\mathbf{CD}^2) = 21.8909$.

Corollary: If each destination has only one antenna ($N = 1$), then the proposed iterative algorithm converges to a unique GNE solution if and only if

$$\rho(\mathbf{CD}^2) < 1,$$

where the matrices $\mathbf{C}$ and $\mathbf{D}$ are specified in (35) and (36), respectively.

IV. Numerical and Simulation Results

In this section, we carry out some numerical and simulation studies to illustrate the performance of the proposed iterative algorithm and compare it with the IWF algorithm. We assume that there are three source-destination pairs ($K = 3$), each source has two transmit antennas ($M = 2$) and each destination has two receive antennas ($N = 2$). Channel coefficients are assumed to be Rayleigh distributed with unit variance. The SINR requirement of each source-destination pair is assumed to be $\gamma_k = 1\,(1 \leq k \leq K)$.

In Figs. 2 and 3, we illustrate the performance of the proposed iterative algorithm with a randomly generated channel condition. Specifically, Fig. 2 shows the required transmission power at each source in each iteration. We observe that with the proposed algorithm, the transmission power of each source converges after 10 iterations, which leads to a GNE solution. With the IWF algorithm, however, the transmission power of each source does not converge and fluctuates periodically. Fig. 3 plots the resulting SINR of each source-destination pair. We can see that with the proposed algorithm, the SINR requirement of each source-destination pair is satisfied after 10 iterations. But with the IWF algorithm, the SINR requirement of the three pairs cannot be satisfied at the same time and the resulting SINRs fluctuate in different iterations.

In Fig. 4, we compare the probability of convergence for the proposed algorithm and the IWF algorithm with different SINR requirements $\gamma_1 = \gamma_2 = \gamma_3 \in$ (0dB, 15dB) over 2000

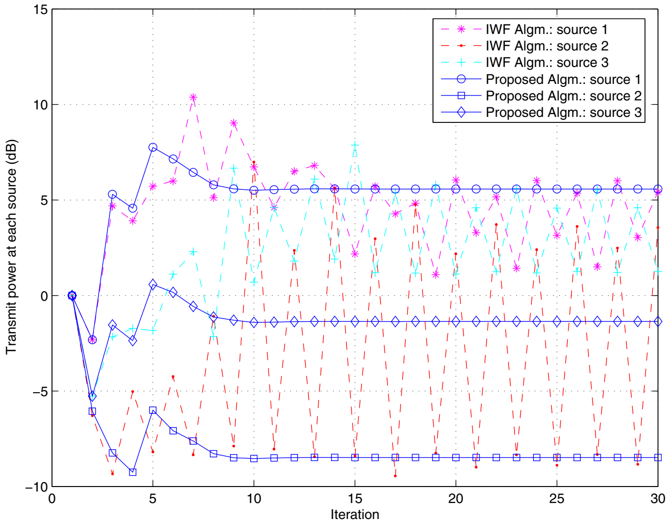

Fig. 2. Transmission power at each source with the proposed algorithm and the IWF algorithm, $K = 3, M = N = 2$.

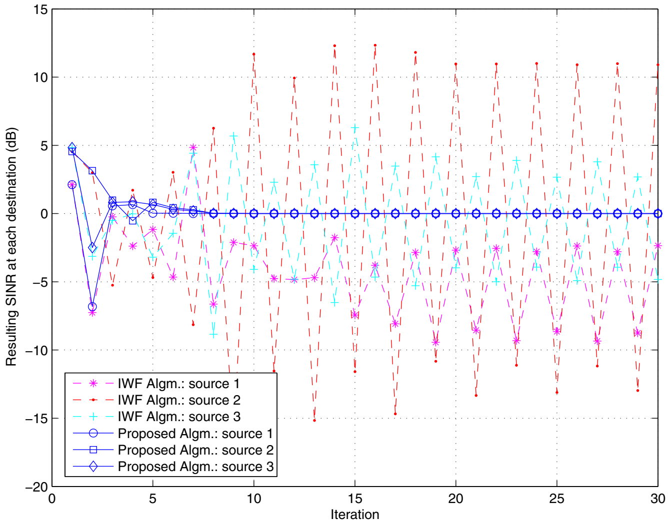

Fig. 3. Resulting SINR at each destination with the proposed algorithm and the IWF algorithm, $K = 3, M = N = 2$.

randomly generated channel realizations. We can see that the convergence probability of the proposed algorithm is much higher than that of the IWF algorithm. For example, with a SINR requirement of 2dB, the probability of convergence is 66% by the IWF algorithm while it is 84% by the proposed algorithm. With a SINR requirement of 5dB, the probability of convergence is improved from 36% to 66% with the proposed algorithm.

V. Conclusion

In this paper, we designed transceiver beamforming vectors for MSMD MIMO wireless networks such that the transmission power of each source is minimized while the SINR requirements of all source-destination pairs are satisfied. We proposed an iterative algorithm to design the transceiver beamforming vectors and discussed the convergence of the algorithm which depends on channel condition. We determined a necessary condition as well as a sufficient condition for the algorithm to converge to a GNE solution. If each destination

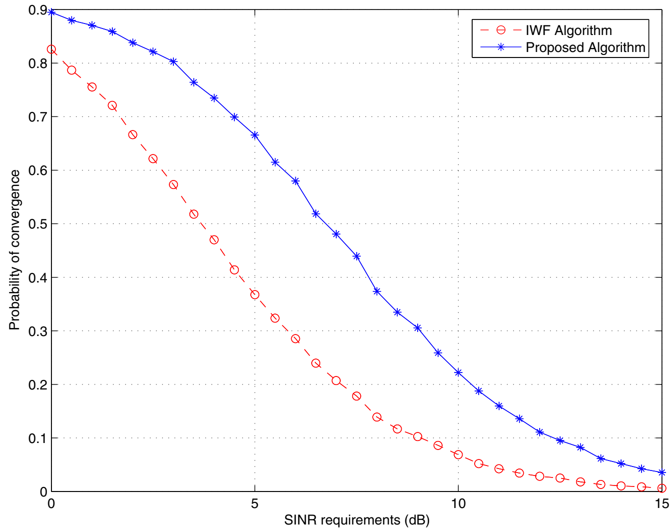

Fig. 4. Probability of convergence of the proposed algorithm and the IWF algorithm, $K = 3, M = N = 2$.

has only one antenna, we obtained a necessary and sufficient condition for the algorithm to converge to a GNE solution. Simulation results show that the proposed iterative algorithm has higher probability of convergence compared to the IWF algorithm. For example, for a system with three source-destination pairs and SINR requirement of 2dB, the probability of convergence is 84% by the proposed algorithm while it is only 66% by the IWF algorithm

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